Non-Gaussianities from cosmic strings in scaling

Christophe Ringeval
Institute of Mathematics and Physics
Centre for Cosmology, Particle Physics and Phenomenology
Louvain University, Belgium

Ann Arbor, 15/05/2011

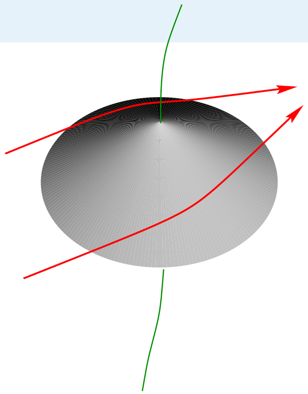
CR: arXiv:1005.4842
M. Hindmarsh, CR, T. Suyama: arXiv:0911.1241, arXiv:0908.0432
A. Fraisse, CR, D. Spergel, F. Bouchet: arXiv:0708.1162

Cosmic strings of various origins

- Line-like remnants of the early universe that should still be present
  - Actively searched in the last 30 years. Yet undetected...
  - Solitons created during cosmological phase transitions [Kibble 76]
  - Cosmologically streched objects from String Theory [Witten 85]
  - Generically formed at the end of inflation [Sarangi 02, Jeannerot 03]
- Prototypical model: Nambu–Goto string networks (one parameter U)
  - Numerical simulations shows that they relax towards a self-similar configuration = scaling
  - Energy density of long strings and loops evolves as radiation/matter instead of $\rho \propto a^{-2}$

$$\rho_\infty \frac{d_{\mathrm{h}}^2}{U}\bigg|_{\mathrm{mat}} = 28.4 \pm 0.9, \qquad \rho_\infty \frac{d_{\mathrm{h}}^2}{U}\bigg|_{\mathrm{rad}} = 37.8 \pm 1.7.$$

Integrated Sachs–Wolfe effect



- Gott–Kaiser–Stebbins effect: conical metric
  - CMB temperature discontinuities

$$\delta T/T_{\mathrm{CMB}} \propto 8\pi G U v$$

- ISW from Nambu–Goto stress tensor + Einstein equations:

[Hindmarsh 95, Stebbins 95]

$$\Theta(\hat{\boldsymbol{n}}) \equiv \frac{\delta T}{T_{\mathrm{CMB}}} = -4GU \int_{\boldsymbol{X}\,\cap\,\boldsymbol{x}_\gamma} \left[\boldsymbol{u}(\hat{\boldsymbol{n}}) \cdot \frac{\boldsymbol{X}_\perp}{\boldsymbol{X}_\perp^2}\right] \left(1 + \hat{\boldsymbol{n}} \cdot \dot{\boldsymbol{X}}\right) \mathrm{d}\sigma$$

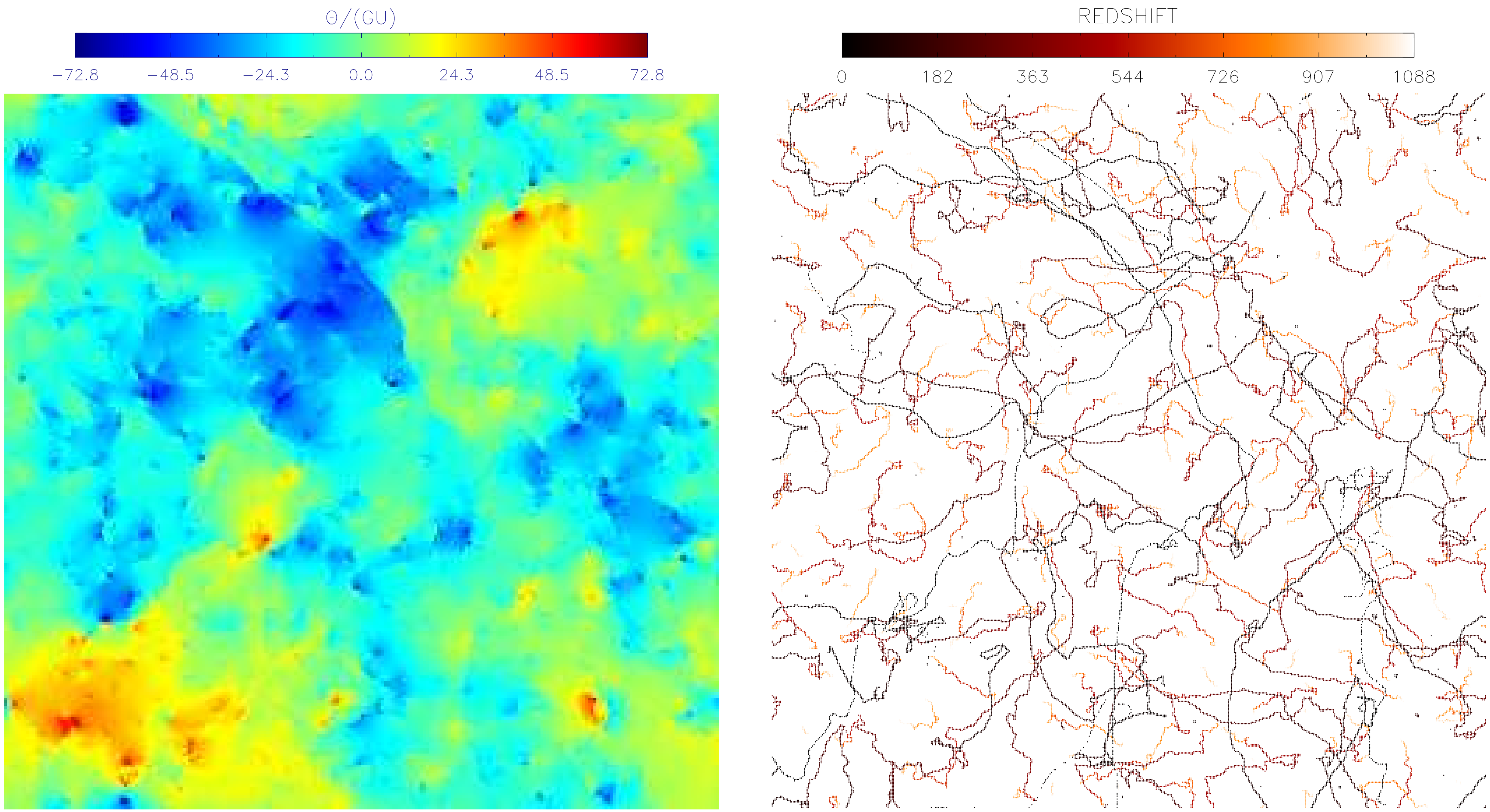
$$\boldsymbol{u} = \dot{\boldsymbol{X}} - \frac{(\hat{\boldsymbol{n}} \cdot \boldsymbol{X}') \cdot \boldsymbol{X}'}{1 + \hat{\boldsymbol{n}} \cdot \dot{\boldsymbol{X}}} \qquad \boldsymbol{X}_\perp \equiv X\hat{\boldsymbol{n}} - \boldsymbol{X}$$

- At small angular scales, in 2D transverse Fourier space ($\boldsymbol{k} \cdot \hat{\boldsymbol{n}} \simeq 0$):

$$\Theta \simeq \frac{8\pi i\, GU}{\boldsymbol{k}^2} \int_{\boldsymbol{X}\,\cap\,\boldsymbol{x}_\gamma} (\boldsymbol{u} \cdot \boldsymbol{k})\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{X}}\, \mathrm{d}\sigma$$

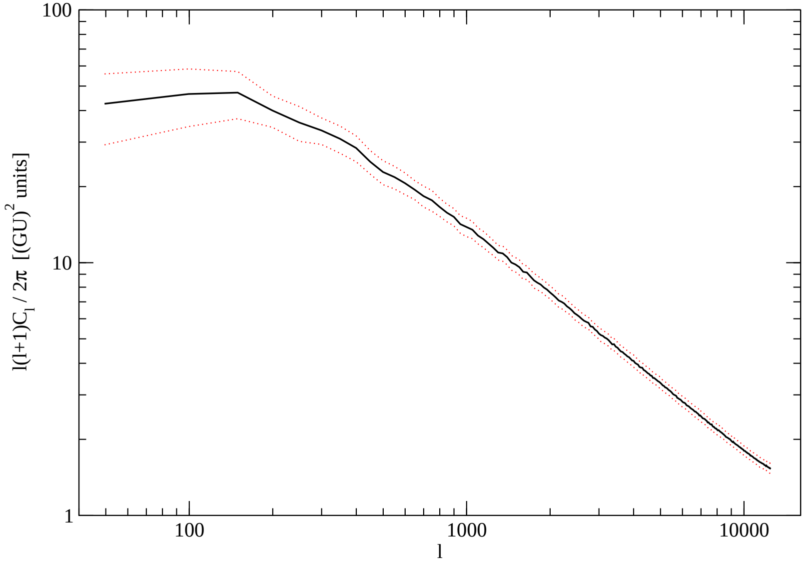
Simulated CMB maps at small angles

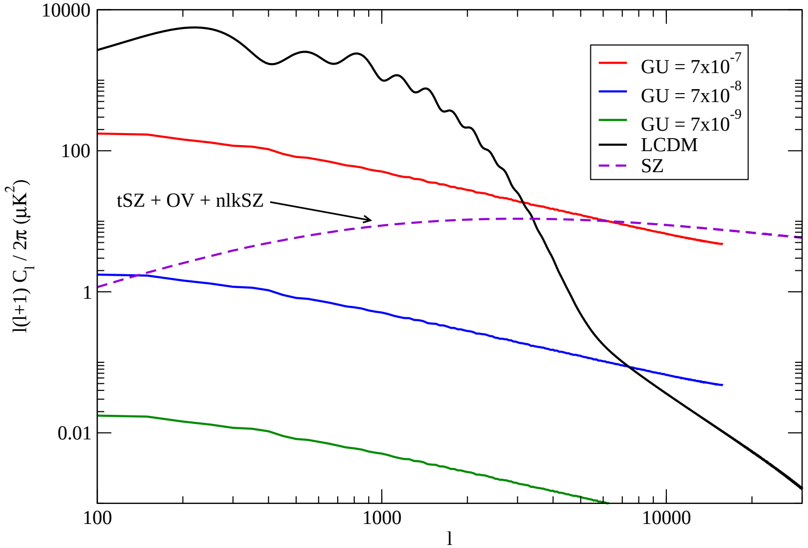
- Temperature anisotropies on a 7.2° field of view (from long strings and loops in scaling)



String effects since last scattering

- String power spectrum dominates at the large multipoles





- Power law behaviour at small scales

$$\ell(\ell+1)\, C_\ell \underset{\ell \gg 1}{\propto} \ell^{-p} \quad \text{with} \quad p = 0.889^{+0.001}_{-0.090}$$

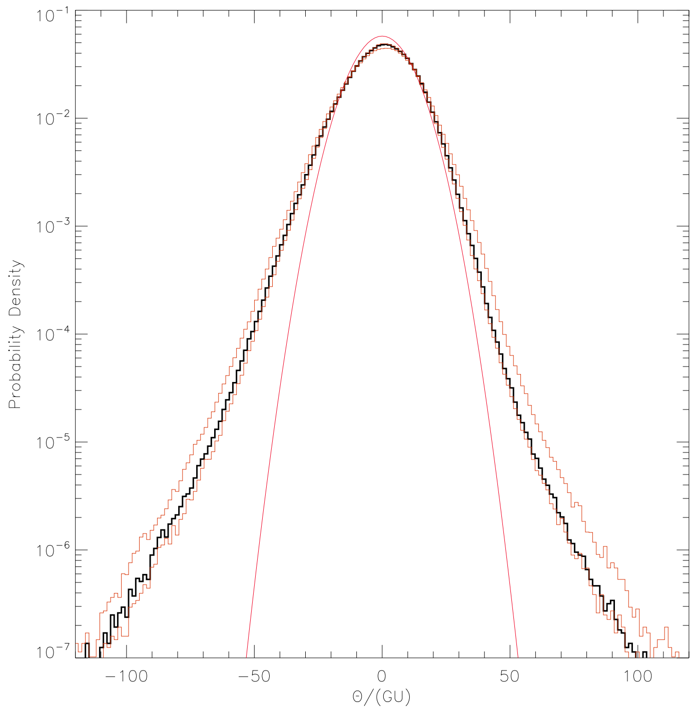
- Recovered with Abelian Higgs strings [Urrestilla 08, Bevis 10] $\Rightarrow GU < 7 \times 10^{-7}$

Basic non-Gaussian estimators

- One-point functions

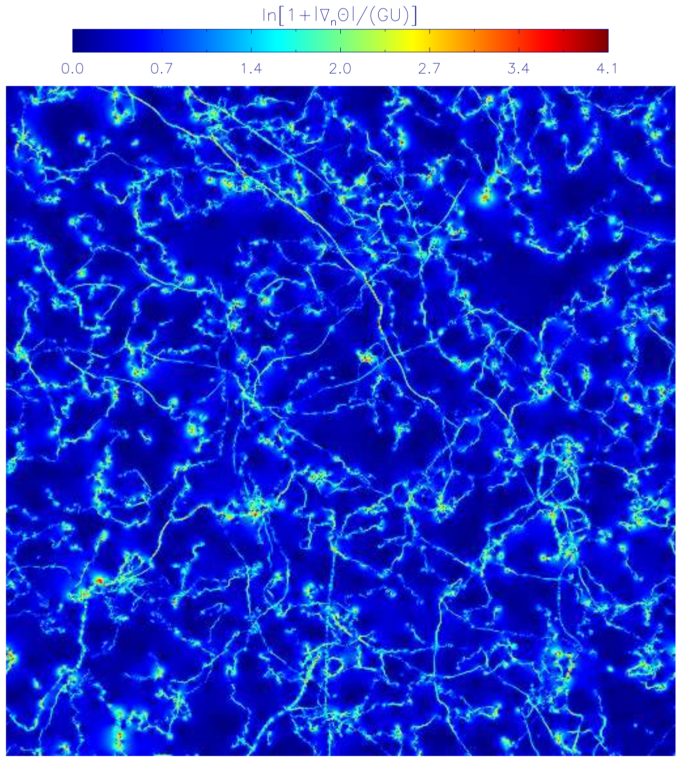
$$g_1 \equiv \left\langle \frac{\overline{(\Theta-\bar{\Theta})^3}}{\sigma^3} \right\rangle \simeq -0.22 \pm 0.12$$

$$g_2 \equiv \left\langle \frac{\overline{(\Theta-\bar{\Theta})^4}}{\sigma^4} \right\rangle - 3 \simeq 0.69 \pm 0.29.$$



- Gradient magnitude

$$|\nabla\Theta| \equiv \sqrt{\left(\frac{d\Theta}{d\alpha}\right)^2 + \left(\frac{d\Theta}{d\beta}\right)^2}$$



Three-point function of the CMB anisotropies

- Non-vanishing skewness $\Rightarrow$ 3-pts function $\neq 0$

$$\langle \hat{\Theta}_{\boldsymbol{k}_1} \hat{\Theta}_{\boldsymbol{k}_2} \hat{\Theta}_{\boldsymbol{k}_3} \rangle = B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)(2\pi)^2 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$$

- From ISW, can be evaluated analytically at small angle (l.c. gauge)

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = i\epsilon^3 \frac{1}{\mathcal{A}} \frac{k_{1A} k_{2B} k_{3C}}{k_1^2 k_2^2 k_3^2} \int \mathrm{d}\sigma_1 \mathrm{d}\sigma_2 \mathrm{d}\sigma_3 \left\langle \dot{X}_1^A \dot{X}_2^B \dot{X}_3^C e^{i\delta^{ab} \boldsymbol{k}_a \cdot \boldsymbol{X}_b} \right\rangle$$

with $\dot{X}_a^A = \dot{X}^A(\sigma_a)$, $a, b \in \{1, 2, 3\}$, $\epsilon = 8\pi G U$

- Assuming $\dot{\boldsymbol{X}}$ and $\acute{\boldsymbol{X}}$ are Gaussian random variables

$$\left\langle C^{ABC} e^{iD} \right\rangle = i \left\langle C^{ABC} D \right\rangle e^{-\langle D^2 \rangle / 2}$$

- Expand everything in terms of two-point correlators: $\sigma_{ab} = \sigma_a - \sigma_b$

$$\left\langle \dot{X}_a^A \dot{X}_b^B \right\rangle = \frac{\delta^{AB}}{2} V(\sigma_{ab}), \left\langle \dot{X}_a^A \acute{X}_b^B \right\rangle = \frac{\delta^{AB}}{2} M(\sigma_{ab}), \left\langle \acute{X}_a^A \acute{X}_b^B \right\rangle = \frac{\delta^{AB}}{2} T(\sigma_{ab})$$

- Integration can be done at large wavenumbers: $\kappa_{ab} \equiv \boldsymbol{k}_a \cdot \boldsymbol{k}_b \gg 1$

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\epsilon^3 \pi c_0 \frac{\bar{v}^2}{\bar{t}^4} \frac{L\hat{\xi}}{\mathcal{A}} \frac{1}{\hat{\xi}^2} \frac{1}{k_1^2 k_2^2 k_3^2} \left[\frac{k_1^4 \kappa_{23} + k_2^4 \kappa_{31} + k_3^4 \kappa_{12}}{(\kappa_{23}\kappa_{31} + \kappa_{12}\kappa_{31} + \kappa_{12}\kappa_{23})^{3/2}} \right]$$

- Leading order sensitive to the (averaged projected) small scales $\sigma \longrightarrow 0$:

$$V(\sigma) \sim \bar{v}^2$$

- T and M dependency appears through

$$\Gamma(\sigma_{ab}) \equiv \left\langle \left[\boldsymbol{X}(\sigma_a) - \boldsymbol{X}(\sigma_b) \right]^2 \right\rangle = \int_{\sigma_b}^{\sigma_a} \mathrm{d}\sigma \int_{\sigma_b}^{\sigma_a} \mathrm{d}\sigma' T(\sigma - \sigma') \sim \bar{t}^2 \sigma_{ab}^2$$

$$\Pi(\sigma_{ab}) \equiv \left\langle \left[\boldsymbol{X}(\sigma_a) - \boldsymbol{X}(\sigma_b)) \right] \cdot \dot{\boldsymbol{X}}(\sigma_b) \right\rangle = \int_{\sigma_b}^{\sigma_a} \mathrm{d}\sigma M(\sigma - \sigma_b) \sim \frac{1}{2} \frac{c_0}{\hat{\xi}} \sigma_{ab}^2$$

String bispectrum comes from expansion

- Proportional to $c_0 \equiv \hat{\xi} \left\langle \ddot{\boldsymbol{X}} \cdot \dot{\boldsymbol{X}} \right\rangle \neq 0$?
- Light cone gauge + FLRW + $\dot{\boldsymbol{X}}$, $\acute{\boldsymbol{X}}$ Gaussian random variables

$$\left\langle \ddot{\boldsymbol{X}} \cdot \dot{\boldsymbol{X}} \right\rangle = \bar{\mathcal{H}} \left(\left\langle \dot{\boldsymbol{X}}^2 \right\rangle \left\langle \acute{\boldsymbol{X}}^2 \right\rangle - \left\langle \dot{\boldsymbol{X}} \cdot \acute{\boldsymbol{X}} \right\rangle^2 \right) = \bar{\mathcal{H}} \bar{v}^2 \bar{t}^2$$

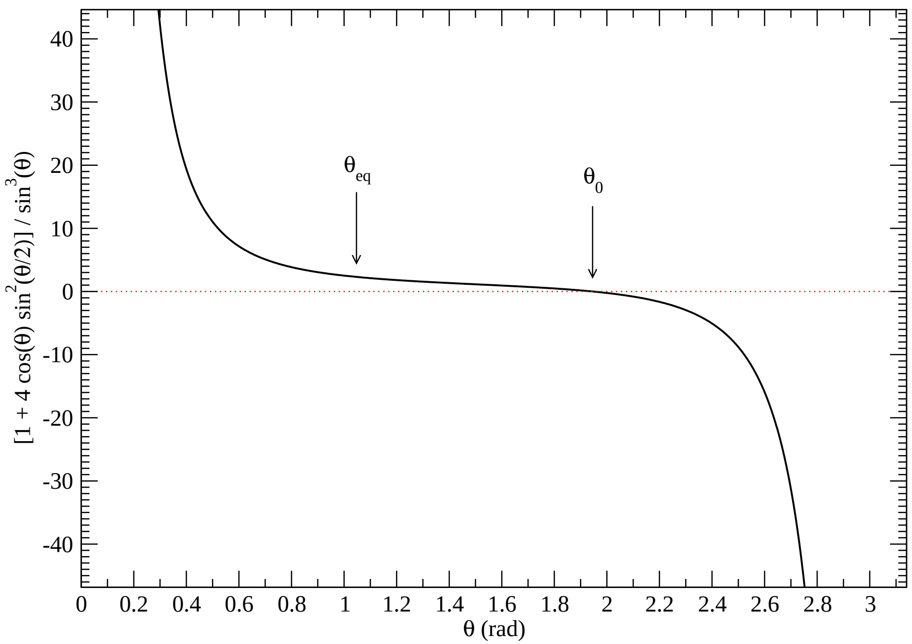
- For $\bar{\mathcal{H}} > 0 \Rightarrow c_0 > 0$: breaking of time reversal invariance
- Gives a negative skewness by integration

Example: isoscele triangle configurations

- Wavenumbers such that $k_1 = k_2 = k$ and $k_3 = 2k\sin(\theta/2)$

$$B_{\ell\ell\theta}(k,\theta) = -\epsilon^3 \pi c_0 \frac{\bar{v}^2}{\bar{t}^4} \frac{L\hat{\xi}}{\mathcal{A}} \frac{1}{\hat{\xi}^2 k^6} \frac{1 + 4\cos\theta\sin^2(\theta/2)}{\sin^3\theta}$$

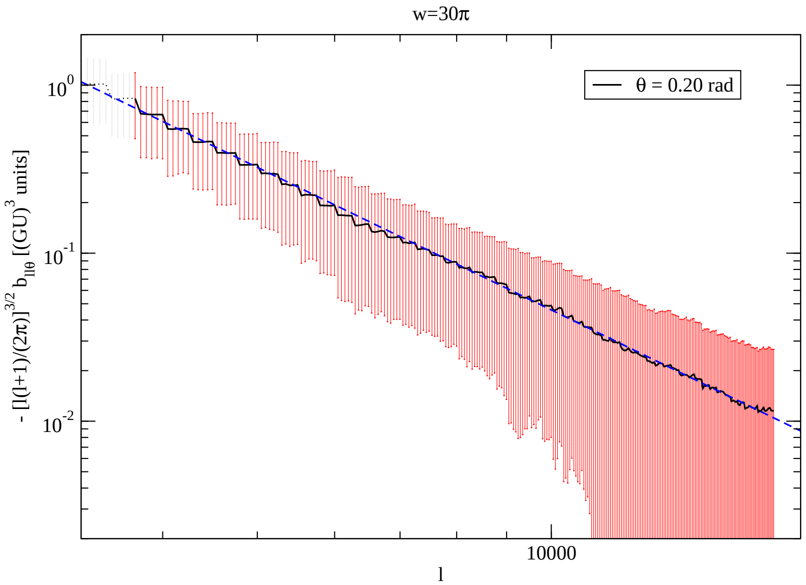
- Amplified on elongated triangles; $\pm$ at $\theta_0 = 2\arccos\dfrac{\sqrt{3-\sqrt{3}}}{2}$

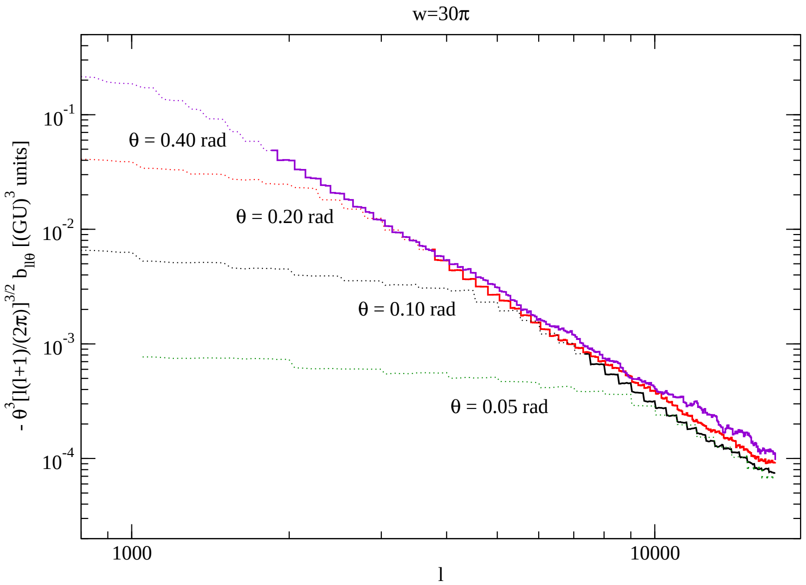


- Estimator [Spergel 99, Aghanim 03, Komatsu 05]: $\Theta_u(\boldsymbol{x}) \equiv \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^2} \hat{\Theta}_{\boldsymbol{k}} W_u(k)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$B_{k_1 k_2 k_3} = \frac{\left\langle \int \Theta_{k_1}(\boldsymbol{x}) \Theta_{k_2}(\boldsymbol{x}) \Theta_{k_3}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \right\rangle}{\int \frac{\mathrm{d}\boldsymbol{p}\mathrm{d}\boldsymbol{q}}{(2\pi)^4} W_{k_1}(p)\, W_{k_2}(q)\, W_{k_3}(|\boldsymbol{p}+\boldsymbol{q}|)}$$

- Power-law and dependency in θ recovered





Four-point function of the CMB anistropies

- Same method with new features: flat directions

$$\left\langle \hat{\Theta}_{\boldsymbol{k}_1} \hat{\Theta}_{\boldsymbol{k}_2} \hat{\Theta}_{\boldsymbol{k}_3} \hat{\Theta}_{\boldsymbol{k}_4} \right\rangle = T(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)(2\pi)^2 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)$$

$$T_{1234} = \frac{\epsilon^4}{\mathcal{A}} \frac{k_{1A} k_{2B} k_{3C} k_{4D}}{k_1^2 k_2^2 k_3^2 k_4^2} \int \mathrm{d}\sigma_1 \mathrm{d}\sigma_2 \mathrm{d}\sigma_3 \mathrm{d}\sigma_4 \left\langle \dot{X}_1^A \dot{X}_2^B \dot{X}_3^C \dot{X}_4^D e^{i\delta^{ab} \boldsymbol{k}_a \cdot \boldsymbol{X}_b} \right\rangle$$

- Trispectrum becomes sensitive to higher order in the correlators

$$\text{Polchinski–Rocha model} \Rightarrow T(\sigma) \simeq \bar{t}^2 - c_1 \left(\frac{\sigma}{\hat{\xi}} \right)^{2\chi}$$

- At large multipoles, sensitives to the string microstructure!
    - $0 < \chi < 1$, $c_1 > 0$
    - NG: power-law exponent of the loop distribution
    - Other strings: related to the mean square velocity

Trispectrum of string induced CMB anisotropies

- Parallelogram configurations [contain $P(k)^2$]: goes as k^{-6}

$$T_0(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \simeq \frac{\pi \epsilon^4 \bar{v}^4}{\bar{t}^2} \frac{L^2}{\mathcal{A} k_1^3 k_2^3 |\sin\theta|}$$

- All the others

$$T_\infty(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \simeq \epsilon^4 \frac{\bar{v}^4}{\bar{t}^2} \frac{L\hat{\xi}}{\mathcal{A}} \left(c_1 \hat{\xi}^2\right)^{-1/(2\chi+2)} f(\chi) g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$

$$f(\chi) = \frac{\pi}{\chi+1} \Gamma\left(\frac{1}{2\chi+2}\right) [4(2\chi+1)(\chi+1)]^{1/(2\chi+2)}$$

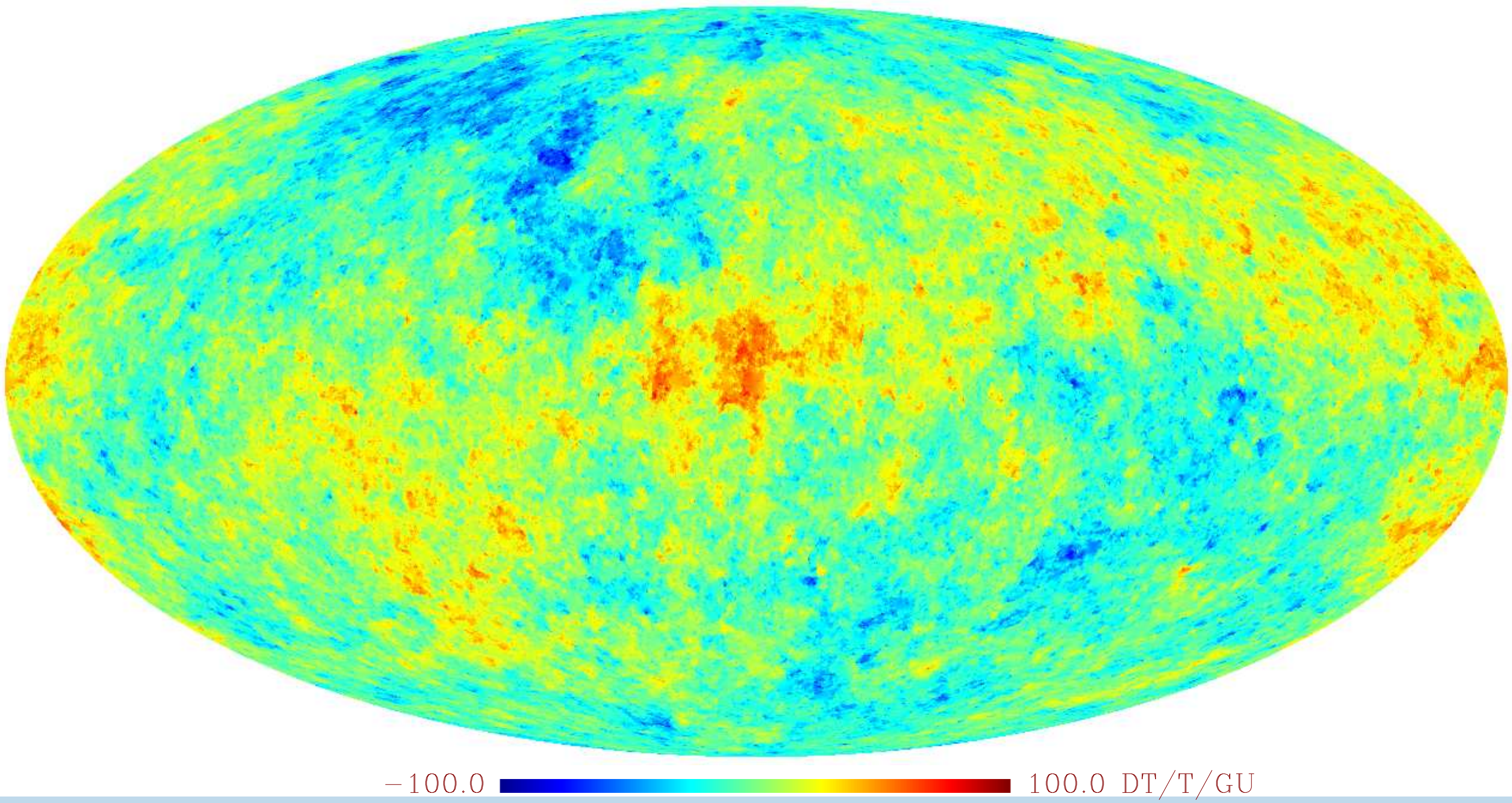
- Geometrical factor scales as k^ρ: $\rho = 6 + 1/(1+\chi)$

$$g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = \frac{\kappa_{12}\kappa_{34} + \kappa_{13}\kappa_{24} + \kappa_{14}\kappa_{23}}{k_1^2 k_2^2 k_3^2 k_4^2} \left[Y^2\right]^{-1/(2\chi+2)}$$

$$Y^2(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \equiv -\kappa_{12} \left(k_3^2 k_4^2 - \kappa_{34}^2\right)^{\chi+1} + \circlearrowleft,$$

Beyond small angles

- Analytical approach extended to larger angles in [Regan 09]
- Simulated full sky map from NG string simulations (challenging)



Conclusion

- Cosmic strings generically imprint non-Gaussianities
- Generated between last-scattering and now $\neq$ primordial type!
- One-point function exhibits a small but non-vanishing skewness and kurtosis
- Non-vanishing bispectrum and trispectrum
  - Decay as $B \propto \ell^{-6}$ and $T \propto \ell^{-6-1/(1+\chi)}$ at large multipoles
  - As for the trispectrum, higher n-point functions are sensitive to the microstructure (fantastic if detected)
  - All amplified on elongated polygons ($+$ some symmetries)
- Test for strings with PLANCK: template matching using analytic shape (c.f. Friday's talks) or simulated maps

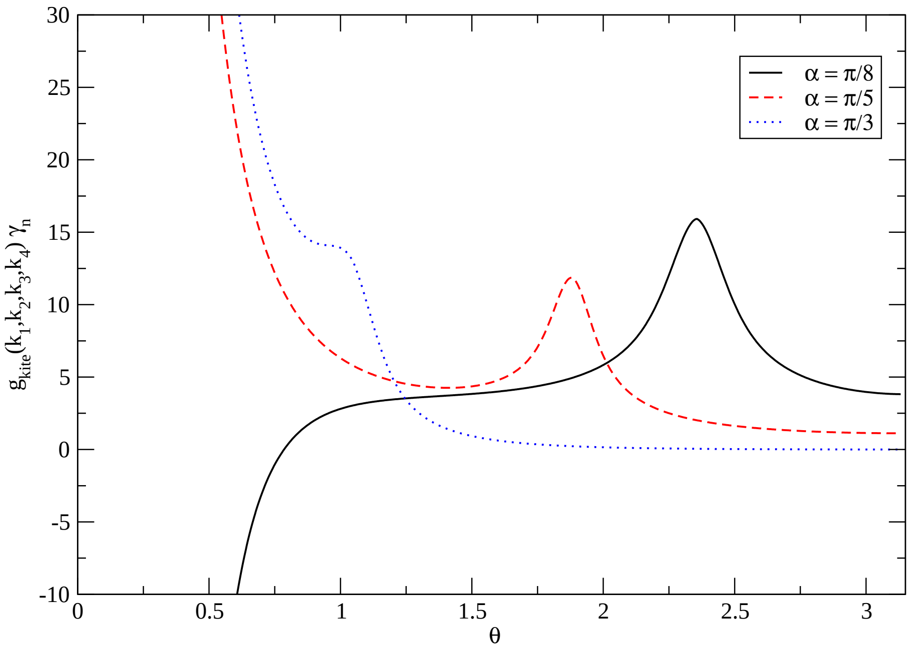
Example: kite quadrilaterals

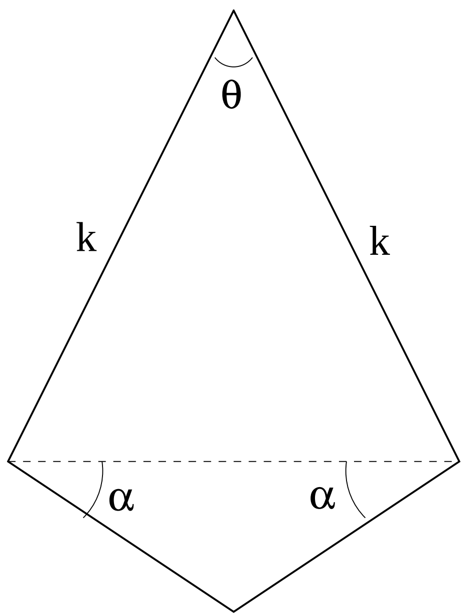
- Geometrical factor for kites: boost on elongated

$$g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = \frac{\cos^2(\alpha)\,[1 - 2\cos(2\alpha)\cos(\theta)]}{\sin^2(\theta/2)} \frac{1}{k^\rho y^{2/(2+2\chi)}(\theta, \alpha)}$$

$$\rho = 6 + \frac{1}{1+\chi}$$

- Bump for parallelograms at $\theta = \pi - 2\alpha$ ($Y^2 = 0$)





Angular dependence of the kite geometrical factor

$$
\begin{aligned}
y^2(\theta,\alpha) = \left[\sin^2(\theta/2)\right]^{1+\chi} \Bigg\{ & 2\sin(\theta/2)\frac{\sin(\alpha-\theta/2)}{\cos\alpha} \times \left[\frac{\cos^2(\alpha-\theta/2)}{\cos^2\alpha}\right]^{1+\chi} \\
& - 2\sin(\theta/2)\frac{\sin(\alpha+\theta/2)}{\cos\alpha}\left[\frac{\cos^2(\alpha+\theta/2)}{\cos^2\alpha}\right]^{1+\chi} \\
& + 4^{1+\chi}\sin^2(\theta/2)\left[\cos^2(\theta/2)\right]^{1+\chi}\frac{\cos(2\alpha)}{\cos^2(\alpha)} \\
& - 4^{1+\chi}\cos(\theta)\left[\sin^2(\theta/2)\tan^2(\alpha)\right]^{1+\chi} \Bigg\}.
\end{aligned}
$$